# Single module identification – multistep method

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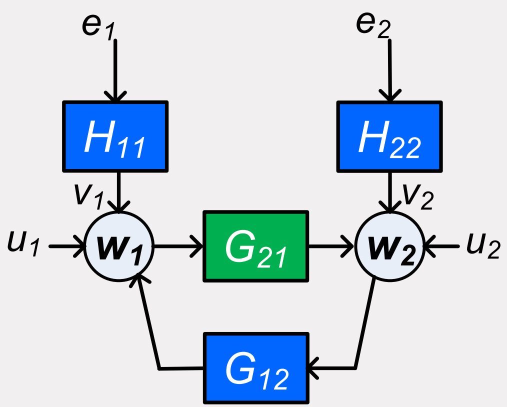
# Local identification – 2 node example

| Method | Transfer | | # u-signals needed |
|---|---|---|---|
| Local Direct [2] | $w \rightarrow w$ | Consistency & ML | 0 |
| Indirect[3] | $u \rightarrow w$ | Consistency | 1 |




Single module identifiability [1]

- Method independent
- Identifiability: no $u$-signal required


[1] Shi, et al. 2021, 2022, 2023
[2] Ramaswamy, et al., TAC, 2021.
[3] Gevers, et al., IFAC, 2018. Hendrickx, et al., TAC, 2019. Bazanella, et al., CDC, 2019.

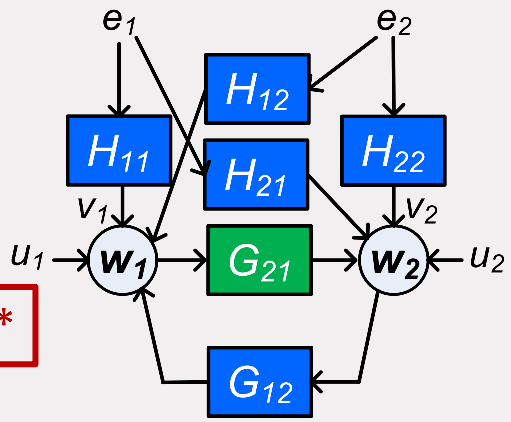



TU/e

# Local identification – confounding variables

| Method | Transfer | | # u-signals needed |
|---|---|---|---|
| Local Direct | $w \rightarrow w$ | Consistency & ML | 2 Conservative |
| Indirect | $u \rightarrow w$ | Consistency | 1 |

MIMO*




*MIMO: Multivariate noise model to model confounding variables as correlated noise

Single module identifiability

- Method independent
- Identifiability: one $u$-signal required

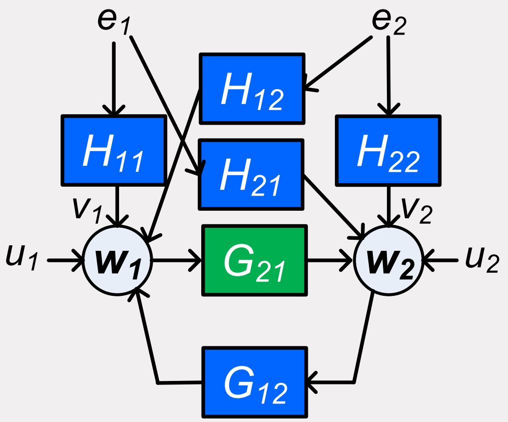
# Question

How can we obtain relaxed conservatism compared to current direct methods in local identification?

- Requires fewer u-signals
- Keep advantages of the current direct methods

# Local identification – confounding variables

| Method | Transfer | | # u-signals needed |
|---|---|---|---|
| Local Direct | $w \rightarrow w$ | Consistency & ML | 2 |
| Indirect | $u \rightarrow w$ | Consistency | 1 |
| Multi-step Least squares [1] | | | 1 |




Full network identification

Single module identifiability

- Method independent
- Identifiability: one $u$-signal

[1] Fonken, et al., Automatica, 2022.

# Multi-step method for full networks [1,2]

**Full network identification** $w = Gw + He + u$

1. **High order ARX model to reconstruct the innovation**
   Estimate $u \rightarrow w$ with high-order ARX and reconstruct $\hat{e}$

← Indirect method

**Rewrite network equation:**

$$w = Gw + (H - I)\hat{e} + Ie + u$$

→ No confounding variables

with $\hat{e}$ an additional measured input

2. **Parametric network model estimate**

$$w_j = \sum_k G_{jk} w_k + \sum_\ell (H_{j\ell} - I_{jj})\hat{e}_\ell + e_j + u_j$$

← Direct method

estimate MISO models for each $j \in \mathcal{L}$, using Weighted Nullspace Fitting [3] → convex

[1] Fonken, et al., Automatica, 2022
[2] Dankers, et al., technical note, 2019
[3] Galrinho, et al., TAC, 2019.

# Local Identification using a multi-step method

1. Selection of nodes

2. High order ARX model to reconstruct the innovation signal

3. Parametric target module estimate

Data Informativity:

*sufficient number of u-signals?*

# Local Identification using a multi-step method

1. **Selection of nodes**

   For a given target mdule $G_{ji}$, select a set of nodes $w_{\mathcal{S}}$ that satisfies the PPL condition

2. **Decompose the nodes**

   Decompose $\mathcal{S}$ in disjunct sets: $\mathcal{S} = \mathcal{Y} \cup \mathcal{U}$, with $j \in \mathcal{Y}$, and $\mathcal{Y}$ contains all nodes in $w_{\mathcal{S}}$ that have a confounding variable with $w_j$

   Consequently:

$$\begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} = \begin{bmatrix} \bar{G} \\ \bar{G}_{\mathcal{U}} \end{bmatrix} \begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} + \left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}} \end{array}\right] \begin{bmatrix} \xi_{\mathcal{Y}} \\ \hline \xi_{\mathcal{U}} \end{bmatrix} + \begin{bmatrix} u_{\mathcal{Y}} \\ \hline u_{\mathcal{U}} \end{bmatrix}$$

$u_{\mathcal{Y}}$ and $u_{\mathcal{U}}$ now refer to external excitation signals in the *immersed* network.

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# Local Identification using a multi-step method

$$\begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} = \begin{bmatrix} \bar{G} \\ \bar{G}_{\mathcal{U}} \end{bmatrix} \begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} + \left[\begin{array}{c|c} \bar{H} & 0 \\ \hline 0 & \bar{H}_{\mathcal{U}} \end{array}\right] \begin{bmatrix} \xi_{\mathcal{Y}} \\ \hline \xi_{\mathcal{U}} \end{bmatrix} + \begin{bmatrix} u_{\mathcal{Y}} \\ \hline u_{\mathcal{U}} \end{bmatrix}$$

3. **Apply the "full network" reasoning to the first part:**

$$w_{\mathcal{Y}} = \bar{G} \begin{bmatrix} w_{\mathcal{Y}} \\ \hline w_{\mathcal{U}} \end{bmatrix} + \bar{H}\xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$

Estimate a high-order ARX model: $u \rightarrow w_{\mathcal{Y}}$ to reconstruct $\xi_{\mathcal{Y}}$

Estimate the $j$-th row of

$$w_{\mathcal{Y}} = \bar{G} \begin{bmatrix} w_{\mathcal{Y}} \\ w_{\mathcal{U}} \end{bmatrix} + (\bar{H} - I)\hat{\xi}_{\mathcal{Y}} + \xi_{\mathcal{Y}} + u_{\mathcal{Y}}$$

# Local Identification using a multi-step method

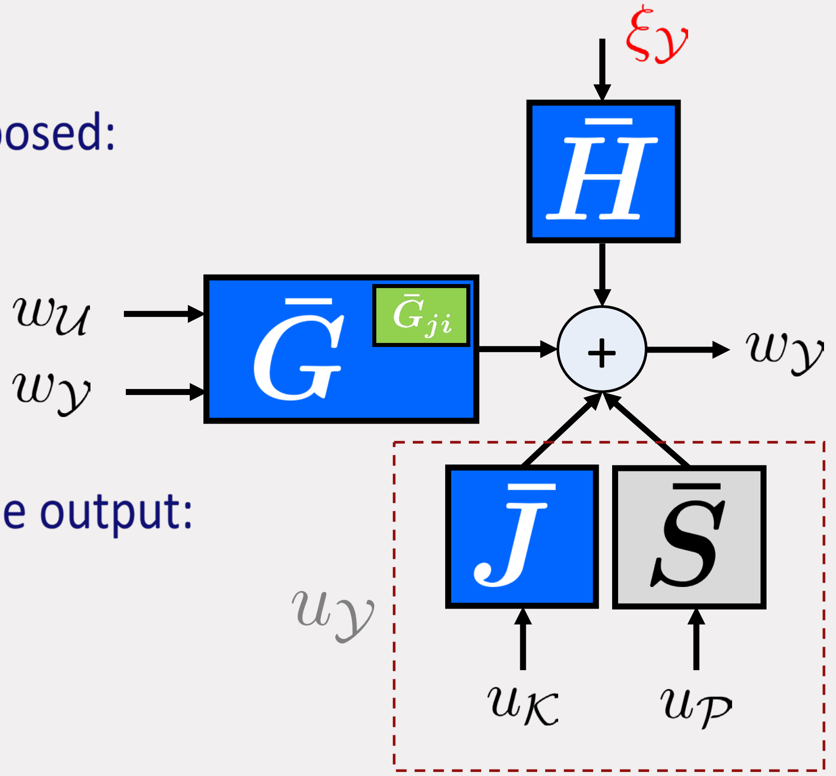
The external signals $u_{\mathcal{Y}}$ can again be decomposed:

$$u_{\mathcal{Y}} = \bar{J}(q)u_{\mathcal{K}} + \bar{S}u_{\mathcal{P}}$$

Dynamic    Known

And for the row related to the target module output:

$$u_j = \sum_{m \in \mathcal{K}_j} \bar{J}_{jm}(q)u_m + u_{\mathcal{P}_j}$$



# Local identification using a multi-step method

## Parametric target module estimate

$$w_j = \sum_{k \in \mathcal{N}_j^-} \bar{G}_{jk} w_k + \sum_{\ell \in \mathcal{Y}} (\bar{H}_{j\ell} - I_{jj}) \hat{\xi}_\ell + \xi_j + \underbrace{\sum_{m \in \mathcal{K}_j} \bar{J}_{jm} u_m + u_{\mathcal{P}_j}}_{u_j}$$

inneighbors in the immersed network

External excitation signals that do **not** contribute to data-informativity: $u_{\mathcal{K}_j}$



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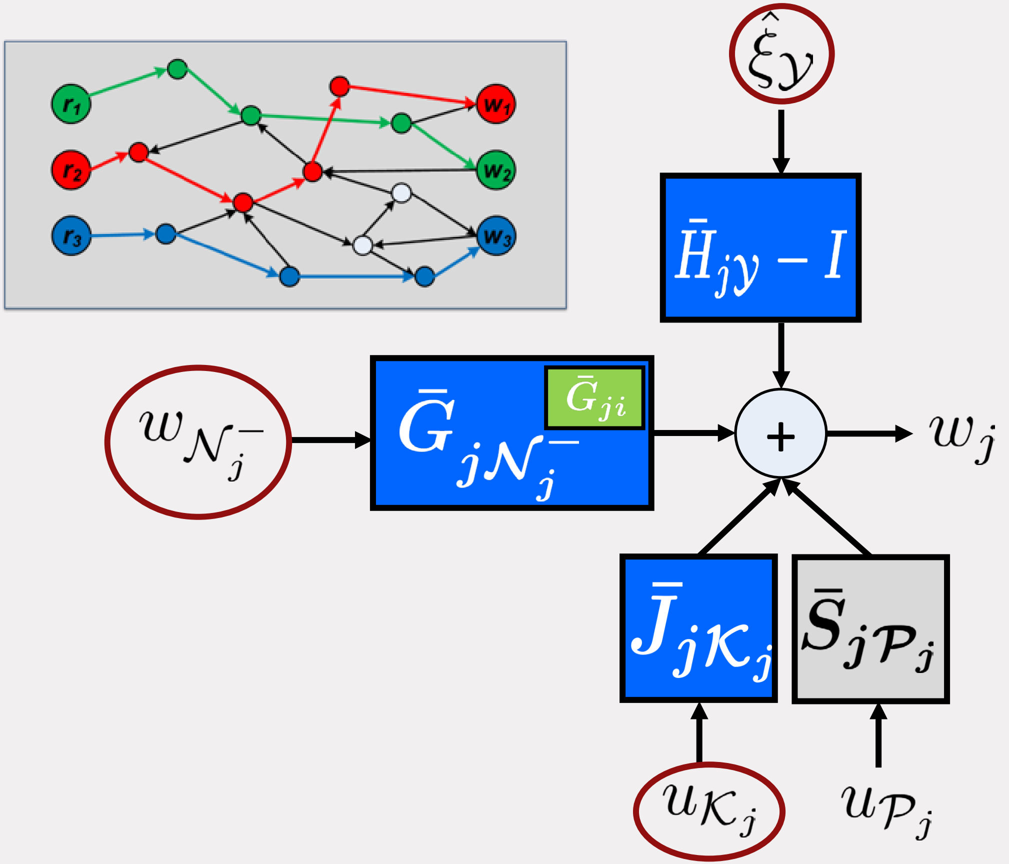
# Data-informativity

$$\Phi_\kappa \succ 0 \qquad \kappa = \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_\mathcal{Y} \\ u_{\mathcal{K}_j} \end{bmatrix}$$

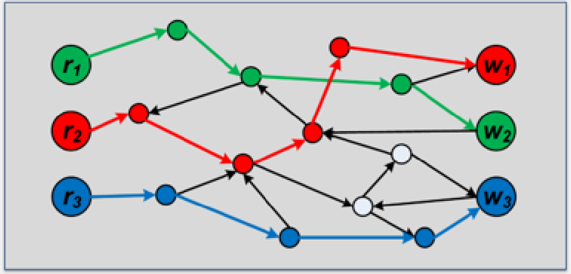
$\kappa$ is persistently exciting holds generically if there are $dim(\kappa)$ vertex disjoint paths from

$$\begin{bmatrix} \xi_\mathcal{S} \\ u_\mathcal{L} \end{bmatrix} \rightarrow \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_\mathcal{Y} \\ u_{\mathcal{K}_j} \end{bmatrix}$$



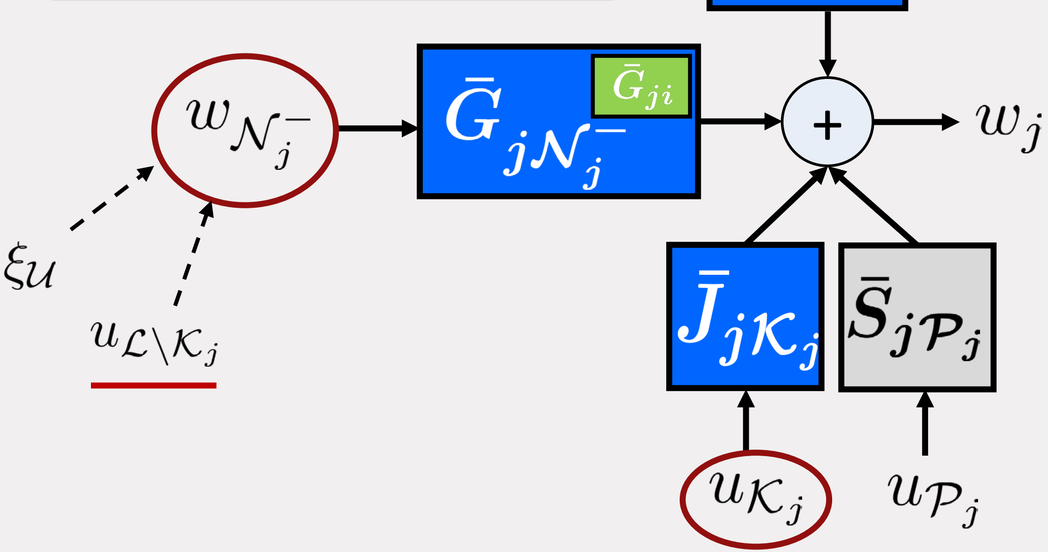
# Data informativity

$$\Phi_\kappa \succ 0 \qquad \kappa = \begin{bmatrix} w_{\mathcal{N}_j^-} \\ \xi_\mathcal{Y} \\ u_{\mathcal{K}_j} \end{bmatrix}$$




$w_{\mathcal{N}_j^-}$ is persistently exciting holds generically if there are $dim(w_{\mathcal{N}_j^-})$ vertex disjoint paths from

$$\begin{bmatrix} \xi_\mathcal{U} \\ u_{\mathcal{L}\backslash\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-}$$

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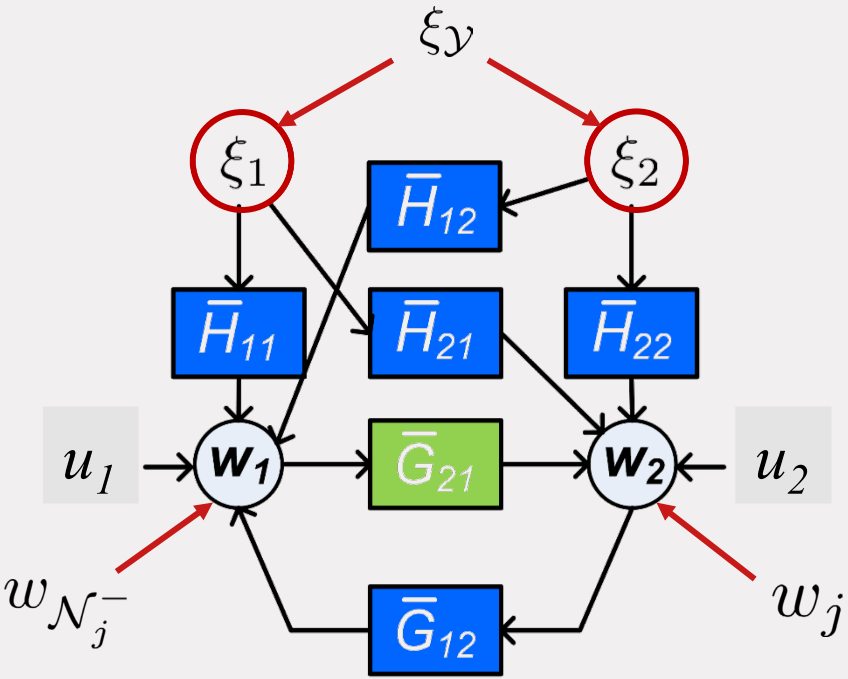
# 2-node example




$$\mathcal{Y} = \{1, 2\}, \mathcal{U} = \emptyset$$

$$u_{\mathcal{L}\setminus\mathcal{K}_j} = u_1, u_2 \quad \notin \mathcal{K}_j = \emptyset$$

$$\Phi_\kappa \succ 0$$

$$\begin{bmatrix} \xi_{\mathcal{U}} \\ u_{\mathcal{L}\setminus\mathcal{K}_j} \end{bmatrix} \rightarrow w_{\mathcal{N}_j^-} \quad \Big\} \quad \begin{bmatrix} \emptyset \\ u_1, u_2 \end{bmatrix} \rightarrow w_1$$

$w_1$ is persistently exciting holds generically if there is 1 vertex disjoint path from $u_1$ or $u_2 \rightarrow w_1$



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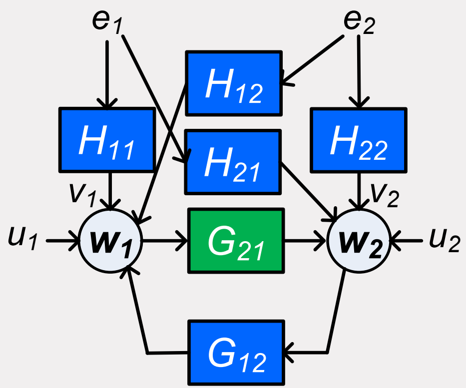
# Local identification

Single module identifiability

- Method independent
- Identifiability: 1 $u$-signal




| Method | Transfer | | # u-signals needed |
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| Indirect | $u \rightarrow w$ | Consistency | 1 |
| Multi-step Least squares | 1. Indirect<br>2. Direct | Consistency & ML? | 1 |

MIMO

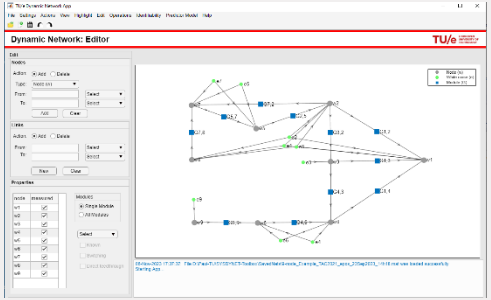
MISO

# Conclusion

Combining indirect and direct methods:

- Requires fewer u-signals than current direct method
- Keeps advantages of the current direct methods
- Parametric estimation with Weighted Null Space Fitting → Convex
- For more details, see [1]

Available in Toolbox www.*sysdynet.net*




[1] S.J.M. Fonken, et al., CDC 2023.